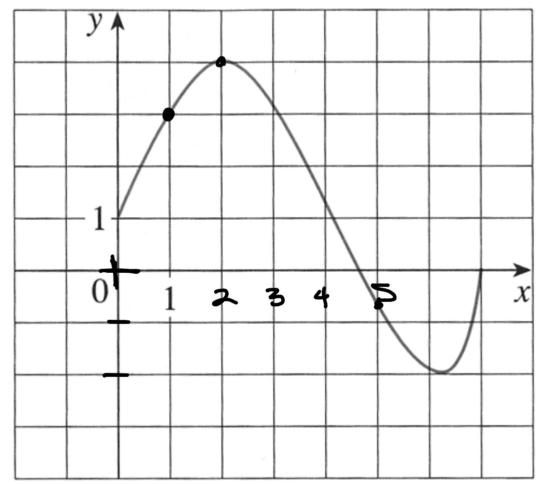
Lecture Notes: 1-1 Four Ways to Represent a Function

Functions can be represented in a variety of ways. Specifically, there are four that we will focus on during this course. They are:

algebraically ($f(x)=x^2+1$), graphically (picture), numerically (table of values), verbally (description of some physical situation.)

Example 1: (Graphically) Interpreting the graph of a function. The graph of a function f is shown below. Find the following:



wants the y-value when $x=1$ and $x=5$

a) $f(1)$ and $f(5)$

$\boxed{f(1)=3}$ $\boxed{f(5)\approx -0.7}$

b) the domain of f ← all x-values lowest → highest

$[0,7]$

c) the range of f ← all y-values lowest → highest

$[-2,4]$

d) For which ~~value~~ of x is $f(x)=4$?

when $\boxed{x=2}$ $\boxed{y=4}$

e) Where if f increasing?

on $[0,2)$ and $(6,7]$

Example 2: (Algebraically) If $f(x)=3x^2-x+2$ find the following. Are (b) and (c) different?

(a) $f(2) = 3(2^2)-2+2$

$= \boxed{12}$

(b) $f(a^2) = 3(a^2)^2-a^2+2$

$= \boxed{3a^4-a^2+2}$

(c) $[f(a)]^2 = [3a^2-a+2]^2 \neq 9a^4-a^2+4$ exponents do NOT distribute over addition.

$= (3a^2-a+2)(3a^2-a+2)$

$= 9a^4-3a^3+6a^2-3a^3+a^2-2a+6a^2-2a+4$

$= \boxed{9a^4-6a^3+13a^2-4a+4}$

input $(a+h)$ for x   input a for x

(d) $\dfrac{f(a+h)-f(a)}{h} = \dfrac{3(a+h)^2-(a+h)+2-(3a^2-a+2)}{h}$

this is called a difference quotient.

$= \dfrac{3(a+h)(a+h)-a-h+2-3a^2+a-2}{h}$

$= \dfrac{3(a^2+2ah+h^2)-h-3a^2}{h}$

$= \dfrac{3a^2+6ah+3h^2-h-3a^2}{h}$

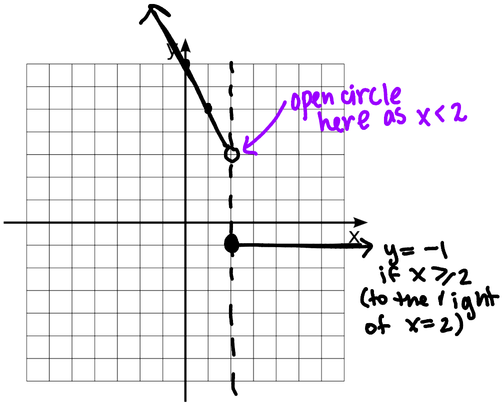
$= \dfrac{6ah+3h^2-h}{h}$

$= \dfrac{h(6a+3h-1)}{h}$

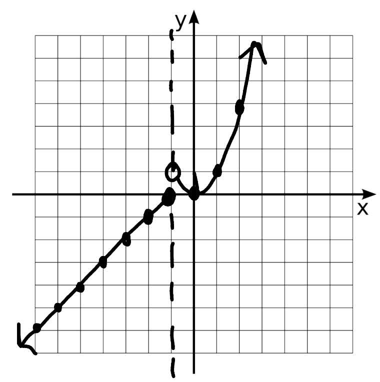
$= \boxed{6a+3h-1}$

Example 3: Graph the following functions. Give the domain and range.

a) $f(x) = \begin{cases} -1 & \text{if } x \geq 2 \\ 7 - 2x & \text{if } x < 2 \end{cases}$



b) $f(x) = \begin{cases} x + 1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$



Domain of a Function:

The **domain** of a function is the set of all possible values of the input. One can find the domain of a function from a picture, but it is also possible to do so from an equation. In many instances it is easier to think about what operations are illegal and leave out the numbers that break these operations. Remember,

these two will be very similar

1. Thou shalt not divide by <u>zero</u>. Set the <u>denominator</u> equal to zero. Leave these numbers out.
2. Thou shalt not square root <u>negatives</u> ← zero is ok. Set the stuff under the radical <u>$\geq$</u> zero and solve. Note, solving polynomial inqualities is not simple.
3. Thou shalt not take the logarithm of <u>neg or 0</u>. Set the stuff inside the logarithm <u>$>$</u> zero and solve. This process is quite similar to # 2.

Example 3: Find the domain of each function. Give the domain using interval notation.

(a) $f(x) = \dfrac{1}{x^4 - 16}$

Leave out where $x^4 - 16 = 0$

$(x^2 - 4)(x^2 + 4) = 0$

$(x+2)(x-2)(x^2+4) = 0$

does not factor

$x = 2, -2$ (exclude these)

$\boxed{D: (-\infty, -2) \cup (-2, 2) \cup (2, \infty)}$

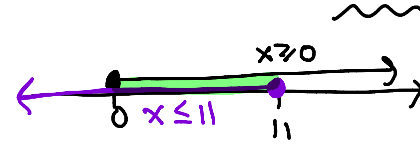
(b) $f(x) = \sqrt{x} + \sqrt{11 - x}$

need $x \geq 0$ and $11 - x \geq 0$

$-x \geq -11$

$x \leq 11$

Picture



to be both, x must be in

$\boxed{[0, 11]}$

Example 4: Find the domain of each function. Give the domain using interval notation.

a) $g(x) = \ln(x^2 - 4)$

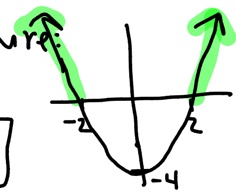
$x^2 - 4 > 0$ ← warning! solving this is not like solving an equation

$(x-2)(x+2) > 0$

sign chart: of $(x-2)(x+2)$: $\oplus$ -2 $\ominus$ 2 $\oplus$

$\boxed{\text{pos. on } (-\infty,-2) \cup (2,\infty)}$

picture:



x^2-4 pos. in green bits.

b) $h(x) = \frac{1}{\sqrt{x^2-5x-6}}$

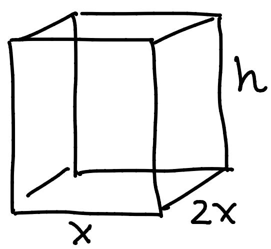
$x^2 - 5x - 6 > 0$

$(x-6)(x+1) > 0$

$\oplus$ -1 $\ominus$ 6 $\oplus$

$\boxed{D: (-\infty,-1) \cup (6,\infty)}$

Example 6: A rectangular storage container with an open top has a volume of 10 m^3. The length of its base is twice the width. Materials for the base cost \$10 per square meter and material for the sides cost \$6 per square meter. Express the cost of materials as a function of the width of the base. Give the domain of the function.



Know: volume is $10\, m^3 \Rightarrow V = lwh \Rightarrow 10 = x \cdot 2x \cdot h$

$\Rightarrow 10 = 2x^2 h$ OR $h = 5/x^2$

cost: $C = 10 * (\text{base area}) + 6 * (\text{sides area})$

$C = 10\,(2x \cdot x) + 6 * (2 \cdot 2xh + 2 \cdot xh)$

$C = 20x^2 + 36xh$ (input this for h)

$C = 20x^2 + 36x \cdot 5/x^2$

$\boxed{C = 20x^2 + 180/x}$

Symmetry

- A function $f(x)$ is called even if $f(-x) = f(x)$. An example is $f(x) = x^2$. Even functions are symmetric about the y-axis.
  ↑ input $(-x)$, simplify, get back to $f(x)$
- A function $f(x)$ is called odd if $f(-x) = -f(x)$. An example is $f(x) = x^3$. Odd functions are symmetric about the origin.
  ↑ input $(-x)$ get $-1 \cdot f(x)$

Example 7: Determine whether the following functions are even, odd, or neither. find $f(-x)$.

a) $g(x) = e^x + 1$

$g(-x) = e^{-x} + 1$

not $g(x)$, not even

not $-g(x)$, not odd

$\boxed{\text{neither}}$

b) $f(x) = 1 + 3x^2 - x^4$

$f(-x) = 1 + 3(-x)^2 - (-x)^4$

$= 1 + 3x^2 - x^4$

$= f(x)$

$\boxed{\text{even}}$

c) $f(x) = \frac{x}{x^2+1}$

$f(-x) = \frac{(-x)}{(-x)^2+1}$

$= \frac{-x}{x^2+1}$

$= -f(x)$

$\boxed{\text{odd}}$

Day 1